Double resonance in cyclotron resonance masers

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In many cyclotron resonance masers a beam of gyrating electrons excites standing waves in resonators. Such waves can be represented as a superposition of forward and backward waves that have opposite Doppler shifts of the operating frequency in the reference frame moving with electrons. Correspondingly, for certain axial wave numbers both forward and backward waves can be in cyclotron resonance with gyrating electrons but at different cyclotron harmonics. The theory describing the interaction between electrons and the resonator field in the case of such a double resonance is developed. It is shown that this double resonance can be beneficial when the device operates at symmetric modes while in the case of operation at nonsymmetric modes it always lowers the efficiency. [S1063-651X(98)02407-6]

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I. INTRODUCTION

Cyclotron resonance masers (CRMs) are known as powerful sources of coherent millimeter- and submillimeter-wave radiation [1,2]. The most advanced group of CRMs are gyrotrons or gyrodevices [3]. Currently, free-running gyrotron oscillators are widely used for electron cyclotron plasma heating and current drive in controlled fusion reactors (see, e.g., Refs. [4] and [5]) and for material processing [6]. Gyroklystrons driven by relativistic electron beams are under development for future (TeV scale) linear colliders [7]. Weakly relativistic gyroklystrons operating at voltages below 100 kV have been developed for radar applications [8–10]. There is also remarkable progress in the development of wideband gyro-traveling-wave tubes (see Ref. [11] and references therein).

In the process of this development practically all of the most important issues for gyrotron operation have been studied theoretically. Some such results in the theory of weakly relativistic gyrotrons can be found in Refs. [1], [12], and [13]. (For recent progress in this field see biennial special issues on high-power microwave generation of the IEEE Transactions on Plasma Science; the latest issue is mentioned in Refs. [5] and [7].) The fundamentals in the theory of relativistic CRMs are given in Ref. [14]. Recent progress is reviewed in Ref. [15]; in particular, Ref. [15] briefly mentions one special case of interaction of gyrating electrons with a resonator field at two cyclotron harmonics simultaneously, in contrast to conventional cyclotron resonance at only one harmonic.

To explain this case let us consider a resonator field with a sinusoidal axial structure $f(z)=\sin(l\pi z/L)$; here l is the axial index and L is the resonator length. This field can be represented as the superposition of two opposite waves with the axial wave numbers $k_z=\pm l\pi/L$. Substituting these axial wave numbers into the cyclotron resonance condition

$$\omega - k_z v_z \simeq s\Omega \tag{1}$$

(where ω and Ω are, respectively, the wave and cyclotron frequencies, v_z is the electron axial velocity, and s is the cyclotron resonance harmonic number), one can readily find that both of the opposite waves may be in resonance with electrons: the forward wave is at the sth harmonic while the opposite one is at the $(s+1)$th or higher harmonic. This case is illustrated in Fig. 1. Such an operation occurs when the ratio of the resonator length L to the wavelength λ relates to the axial index l as

$$\frac{L}{\lambda}=\frac{2s+1}{2}\,\beta_{z0} l. \tag{2}$$

Here β_{z0} is the initial axial velocity of electrons normalized to the speed of light. The corresponding Doppler upshift of the operating frequency with respect to the cyclotron frequency is

$$\frac{\omega}{\Omega}=\frac{2s+1}{2}. \tag{3}$$

Note that the condition of the double resonance given by Eq. (2) can be realized in the case of fast waves ($k_z=l\pi/L < \omega/c$) only when the electron axial velocity is large enough:

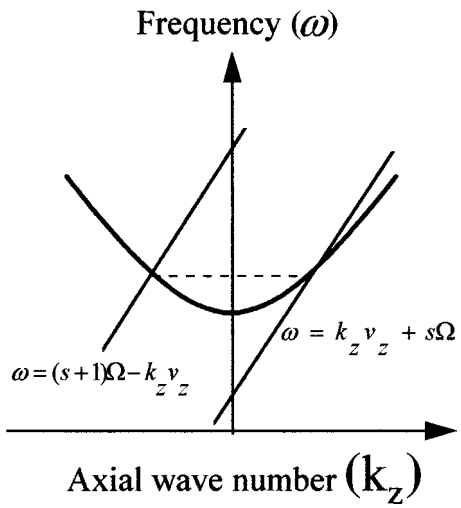

FIG. 1. Dispersion diagram illustrating the double resonance of one resonator mode formed by two traveling waves with an electron beam at two cyclotron harmonics.

$$\beta_{z0} > \frac{1}{2s+1}. \tag{4}$$

For the simplest case of the forward wave interaction at the fundamental harmonic ($s=1$) Eqs. (2)–(4) are reduced, respectively, to $L/\lambda = (\frac{3}{2})\beta_{z0} l$, $\omega/\Omega = \frac{3}{2}$, and $\beta_{z0} > \frac{1}{3}$.

It is expedient to recall that a gyrating electron can lose all of its kinetic energy in the process of interacting with an electromagnetic wave when the axial phase velocity of this wave normalized to the speed of light is equal to [15]

$$\left[\frac{(1+\alpha^2)(\gamma_0-1)}{\gamma_0+1}\right]^{1/2}.$$

Here γ_0 is the initial energy of the electron normalized to the rest energy and $\alpha = p_{\perp 0}/p_{z0}$ is the ratio of the initial components of the electron momentum. As one can easily find, this condition of complete deceleration of one electron is consistent with the condition of the double resonance, $L/\lambda = (\frac{3}{2})\beta_{z0} l$, when $\alpha_{\rm opt}^2 = 2 + 3/\gamma_0$ and $\gamma_0 > \frac{3}{2}$. The latter implies an operation at voltages above 250 kV. Note that the condition of the double resonance at fast waves, $\beta_{z0} > \frac{1}{3}$, for $\alpha = \alpha_{\rm opt}$ means $\gamma_0 > \frac{4}{3}$, i.e., voltages above 150 kV. For $\beta_{z0} > \frac{1}{3}$, Eq. (2) gives $L/\lambda > l/2$, which in principle can be realized at any axial index l. Note that these simple estimates allow one also to evaluate the number of electron orbits in the interaction space, $N = \Omega T/2\pi$, where $T = L/v_{z0}$ is the electron transit time through the resonator. As follows from Eqs. (2) and (3), $N = l$, i.e., under conditions of the double resonance the number of electron orbits is equal to the axial index of the resonator mode.

This simple qualitative analysis indicates some specific features of the double resonance at different harmonics that may occur in CRMs in the process of interacting with one resonator mode. The theory of this phenomenon is developed in this paper, which is organized as follows. In Sec. II a general formalism is described (the derivation of corresponding equations is given in the Appendix). In Sec. III the results are presented and discussed. Section IV contains the summary.

II. GENERAL FORMALISM

As will be shown in the Appendix, the electron motion in the case of the double resonance at two different cyclotron harmonics can be described by the following equations:

$$\frac{dp_\perp}{dz} = \frac{1}{3\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} - 2J_2'(\rho)L_2 e^{i\vartheta_2}]\}, \tag{5}$$

$$\frac{d\theta}{dz} - \frac{\mu}{p_z} = \frac{1}{3\kappa\beta_z p_\perp} \mathrm{Re}\{A[[\rho J_1'(\rho)]' L_1 e^{i\vartheta_1} - [\rho J_2'(\rho)]' L_2 e^{i\vartheta_2}]\}, \tag{6}$$

$$\frac{dp_z}{dz} = \frac{h\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} + J_2'(\rho)L_2 e^{i\vartheta_2}]\}, \tag{7}$$

$$\frac{d\gamma}{dz} = \frac{\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} - J_2'(\rho)L_2 e^{i\vartheta_2}]\}. \tag{8}$$

Here, the components of electron momentum are normalized to mc, the axial coordinate z is normalized to ω/c, the resonator field amplitude A is normalized to $e/mc\omega$, κ and h are, respectively, transverse $k_\perp$ and axial k_z wave numbers normalized to $k = \omega/c$, μ is the ratio of the nonrelativistic cyclotron frequency to the operating frequency, $\rho = k_\perp a$ is the Larmor radius of electrons normalized to $k_\perp$ (ρ can also be redefined as $\kappa p_\perp/\mu$), slowly variable phases $\vartheta_1 = \omega t - k_z z - \theta$ and $\vartheta_2 = \omega t + k_z z - 2\theta$ describe the departure of the forward and opposite waves, respectively, from the exact cyclotron resonance, and θ is the electron gyrophase. Factors L_1 and L_2, to be discussed in the Appendix, describe the transverse structure of the Lorentz force with which, respectively, the forward and opposite waves act on electrons. In the case of a thin annular electron beam interacting with the $\mathrm{TE}_{m,p}$ mode of a cylindrical cavity

$$L_s = J_{m\mp s}(k_\perp R_g)\bar{e}^{\,i(m\mp s)\psi}. \tag{9}$$

Here R_g and ψ are polar coordinates of electron guiding centers. Certainly, one from Eqs. (5), (7), and (8) can be eliminated since $\gamma^2 = 1 + p_\perp^2 + p_z^2$. We presented above all three just for the completeness of our consideration. In the absence of electron velocity spread at the entrance $p_\perp$, p_z, and γ obey the following boundary conditions: $p_\perp(0) = \gamma_0\beta_{\perp 0}$, $p_z(0) = \gamma_0\beta_{z0}$, $\gamma(0) = \gamma_0$. The boundary condition for the gyrophase θ in the gyromonotron is $\theta(0) = \theta_0$ where the initial gyrophase θ_0 is homogeneously distributed from 0 to 2π. In any gyrodevice with electron prebunching the gyrophase at the entrance to the output cavity depends on the prebunching history. This issue will be specified below.

The interaction efficiency can be determined as

$$\eta = \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{1}{2\pi}\int_0^{2\pi}\frac{\gamma_0-\gamma(L)}{\gamma_0-1}\,d\theta_0\right\}d\psi. \tag{10}$$

Here the averaging over θ_0 means the averaging over initial gyrophases in one beamlet, the averaging over ψ means the averaging over all beamlets having different azimuthal coordinates of guiding centers.

Using Eq. (9) one can introduce the normalized amplitude $F = iA|L_1|/2\kappa$ and phase $\vartheta_1' = \vartheta_1 - (m-1)\psi$. (Without a lack of generality we will consider the TE mode rotating in the same azimuthal direction as the electron gyration. In the case of a symmetric $\mathrm{TE}_{0,p}$ mode this does not play any role.) Correspondingly, in Eqs. (5)–(8) the ratio $|L_2|/|L_1|$ can be denoted as R and the phase of the last terms in the right-hand sides of Eqs. (5)–(8) can be rewritten as

$$\theta_2' = \vartheta_2 - (m-2)\psi = 3k_z z - \omega t + 2\vartheta_1' + m\psi = 3h\beta_z - \int_0^z \frac{dz'}{\beta_z} + 2\vartheta_1' + m\psi. \tag{11}$$

Equation (11) shows that in the case of operating at symmetric ($m=0$) modes the second averaging in Eq. (10) is redundant since all beamlets are in identical conditions while for operation at nonsymmetric modes it can be very important.

In new notations Eqs. (5)–(8) can be rewritten as

$$\frac{dp_\perp}{dz}=\frac{2}{3\beta_z}\,\mathrm{Re}\{F[J_1'(\rho)e^{i\vartheta_1}-2RJ_2'(\rho)e^{i\vartheta_2}]\},\quad(12)$$

$$\frac{d\theta}{dz}-\frac{\mu}{p_z}=\frac{2}{3\beta_z p_\perp}\,\mathrm{Re}\{-iF[[\rho J_1'(\rho)]'e^{i\vartheta_1}-R[\rho J_2'(\rho)]'e^{i\vartheta_2}]\},\quad(13)$$

$$\frac{dp_z}{dz}=h\,\frac{\beta_\perp}{\beta_z}\,\mathrm{Re}\{F[J_1'(\rho)e^{i\vartheta_1}+J_2'(\rho)Re^{i\vartheta_2}]\},\quad(14)$$

$$\frac{d\gamma}{dz}=\frac{\beta_\perp}{\beta_z}\,\mathrm{Re}\{F[J_1'(\rho)e^{i\vartheta_1}-RJ_2'(\rho)e^{i\vartheta_2}]\}.\quad(15)$$

Here, the primes for ϑ_1 and ϑ_2 are omitted. To make this set of equations self-consistent we should replace Eq. (13) for θ by the following equation for ϑ_1:

$$\frac{d\vartheta_1}{dz}=\frac{1}{\beta_z}\left\{\delta+\frac{1}{3}\left(1-\frac{\beta_z}{\beta_{z0}}\right)+\mu\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)\right\}+\frac{2}{3\beta_z p_\perp}\,\mathrm{Re}\{iF[[\rho J_1'(\rho)]'e^{i\vartheta_1}-R[\rho J_2'(\rho)]'e^{i\vartheta_2}]\},\quad(16)$$

where $\delta=1-h\beta_{z0}-\mu/\gamma_0$ is the initial mismatch of the cyclotron resonance for the forward wave and take into account that the relation between ϑ_1 and ϑ_2 is given by Eq. (11).

We can also go one step further and, assuming that the argument ρ in Bessel functions is small enough, expand these functions as $J_n(\rho)\simeq(1/n!)(\rho/2)^n$. Then, after introducing $F'=F/2$, $R'=\kappa R$ (primes are omitted below), we reduce Eqs. (12), (14), (15), and (16) to

$$\frac{dp_\perp}{dz}=\frac{1}{\beta_z}\,\mathrm{Re}\left[F\left(\frac{2}{3}\,e^{i\vartheta_1}-R\beta_\perp e^{i\vartheta_2}\right)\right],\quad(17)$$

$$\frac{dp_z}{dz}=h\,\frac{\beta_\perp}{\beta_z}\,\mathrm{Re}\left[F\left(e^{i\vartheta_1}+\frac{3}{4}\,R\beta_\perp e^{i\vartheta_2}\right)\right],\quad(18)$$

$$\frac{d\gamma}{dz}=\frac{\beta_\perp}{\beta_z}\,\mathrm{Re}\left[F\left(e^{i\vartheta_1}-\frac{3}{4}\,R\beta_\perp e^{i\vartheta_2}\right)\right].\quad(19)$$

$$\frac{d\vartheta_1}{dz}=\frac{1}{\beta_z}\left\{\delta+\frac{1}{3}\left(1-\frac{\beta_z}{\beta_{z0}}\right)+\frac{2}{3}\left(1-\frac{\gamma_0}{\gamma}\right)+\frac{1}{p_\perp}\,\mathrm{Re}\left[iF\left(\frac{2}{3}\,e^{i\vartheta_1}-R\beta_\perp e^{i\vartheta_2}\right)\right]\right\}.\quad(20)$$

These equations contain the same three parameters that describe a single-resonance interaction with one resonator mode (see, e.g., Ref. [1]): the rf field amplitude F, the cyclotron resonance mismatch δ, and the resonator length L. They also include the parameter R, which determines the ratio of coupling coefficients to the opposite and forward waves, and the initial components of the electron velocity, $\beta_{\perp 0}$ and β_{z0} [$\gamma_0=(1-\beta_{\perp 0}^2-\beta_{z0}^2)^{-1/2}$]. Note that, as follows from the condition of double resonance, $h\approx 1/3\beta_{z0}$ and L is determined, in accordance with Eq. (2), by β_{z0} and the axial index of the mode, l.

III. RESULTS

The double resonance interaction was studied for a 500-kV electron beam with the orbital-to-axial velocity ratio, $\alpha=1.5$. The interaction with modes having two, three, and four axial variations ($l=2$, 3, and 4) was analyzed for the case of a nonprebunched electron beam and the case of ballistic prebunching. The latter implies a weak modulation in electron energies in the first resonator followed by a long drift section in which this modulation causes significant orbital phase bunching. Correspondingly, the boundary condition to Eq. (19) for the electron energy was written as $\gamma(0)=\gamma_0$ and to Eq. (2) for the phase as

$$\vartheta_1(0)=\vartheta_0+q\,\sin\,\vartheta_0,\quad(21)$$

where the bunching parameter q (see, e.g., Ref. [12]) in the case of a nonprebunched beam is equal to zero. We analyzed the operation at both symmetric ($m=0$) and nonsymmetric ($m\neq 0$) modes. The interaction length was determined by Eq. (2) for $s=1$ and β_{z0} defined by V_b and α specified above: $L/\lambda\simeq 0.72l$. The efficiency given by Eq. (10) was calculated as a function of the field amplitude F for different values of the ratio of coupling coefficient, R, and for each F and R the cyclotron resonance mismatch δ was optimized for maximizing the efficiency.

Results are presented in Figs. 2 and 3, which correspond, respectively, to two and three axial variations ($l=2$ and 3) of the resonator mode. Parts (a), (c), and (e) in these two figures correspond to operation at the symmetric mode while cases (b), (d), and (f) correspond to nonsymmetric modes [the latter set of figures implies double averaging in Eq. (10) while the former corresponds to the single averaging]. Parts (a) and (b) correspond to the nonprebunched electron beam, parts (c) and (d) correspond to the bunching parameter $q=1$, and parts (e) and (f) correspond to $q=1.8$. (This value is close to the optimum for a one-cavity prebunching for operation at the fundamental resonance; see, e.g., Refs. [1] and [12].)

The results presented in Figs. 2 and 3 demonstrate that in the case of operation at symmetric modes the double resonance can improve the efficiency when the coupling to the opposite wave is not too strong ($R=0.2-0.4$). This effect is better pronounced at small values of the bunching parameter ($q=0$ and 1.0). For the optimum bunching parameter ($q=1.8$) the maximum efficiency for the nonzero R's is approximately the same or smaller than for $R=0$. The maximum efficiency of interaction of the prebunched ($q=1.8$) electron beam with modes having two and three axial variations is, respectively, 38% and 45%. We also studied the operation at the mode with four axial variations and found that in this case the maximum efficiency is 46%. Since the maximum efficiency in the last two cases ($l=3$ and 4) was approximately the same, we did not analyze the interaction with modes having a larger ($l>4$) number of axial variations. Relatively small changes in the efficiency due to the additional resonance can be explained by the fact that, as our calculations showed, the parameters optimal for the efficient

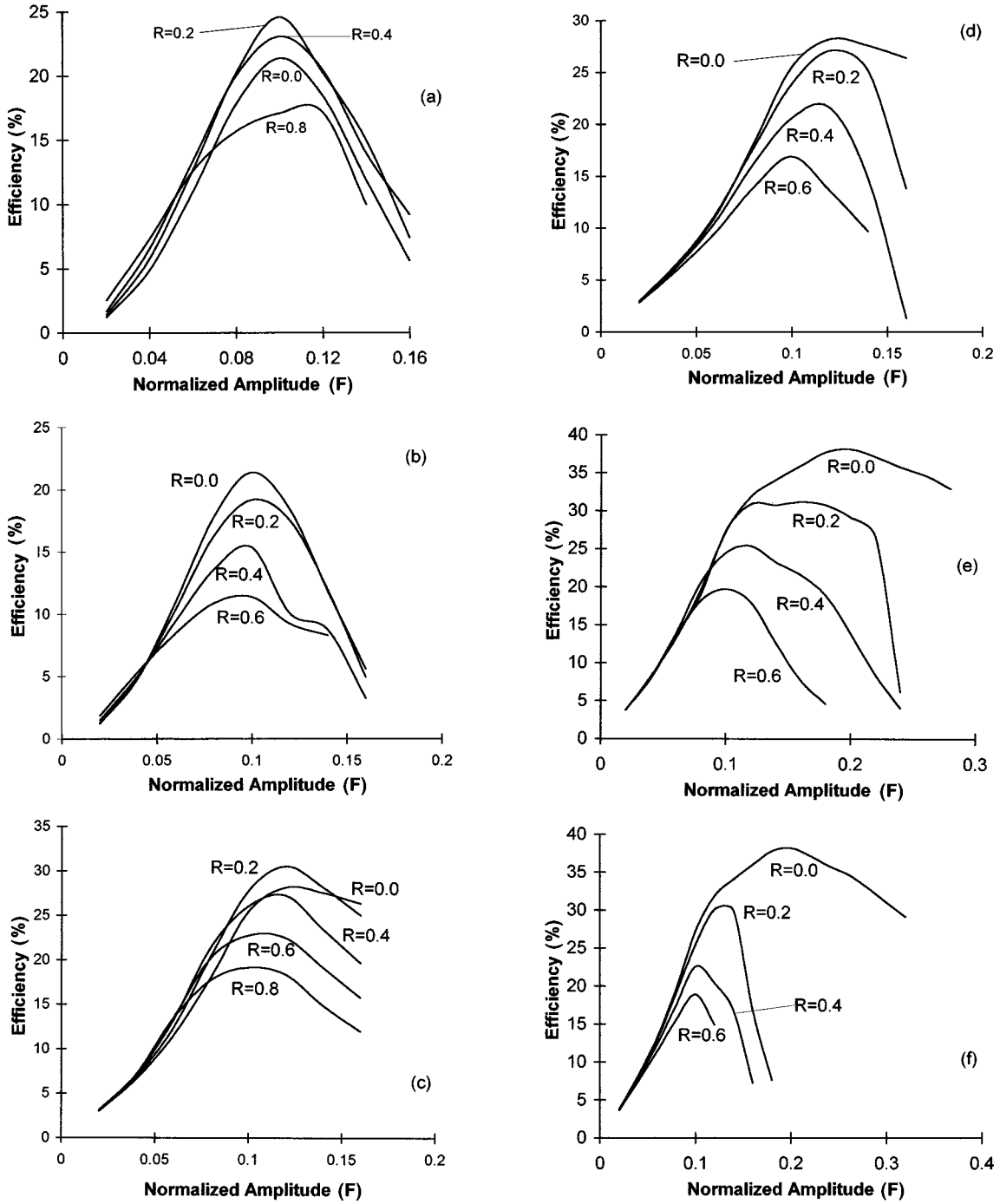

FIG. 2. Operation at a mode with two axial variations: (a), (c), and (e) correspond to the symmetric $TE_{m,p}$ mode ($m=0$); (b), (d), and (f) correspond to nonsymmetric ($m\neq 0$) modes. The bunching parameters for (a) and (b), (c) and (d), and (e) and (f) are equal to 0. 1.0, and 1.8, respectively.

interaction with the backward wave differ significantly from the parameters optimal for the forward wave interaction.

The double resonance in the case of operation at nonsymmetric modes, as follows from cases (b), (d), and (f) shown in Figs. 2 and 3, always only reduces the efficiency in comparison with a single resonance interaction ($R=0$). Moreover, at large amplitudes of the resonator field it causes the appearance of reflected particles since some electrons lose their axial momentum completely in the process of deceleration. Certainly, the absolute values of the efficiency can be enhanced when two or more prebunching cavities are used (see, e.g., Ref. [16]).

The effect of electron velocity spread on the efficiency of double resonance interaction was also studied. We considered the case when a monoenergetic electron beam has a spread in pitch ratios α's, which is typical for the beams formed by magnetron injector electron guns. This spread was modeled by a top hat distribution

$$f(\alpha)=\begin{cases}\text{const} & \text{for } \alpha_0-\sqrt{3}\Delta\alpha\leqslant\alpha\leqslant\alpha_0+\sqrt{3}\Delta\alpha\\ 0 & \text{for } \alpha<\alpha_0-\sqrt{3}\Delta\alpha,\quad \alpha>\alpha_0+\sqrt{3}\Delta\alpha,\end{cases}$$

where the factor $\sqrt{3}$ ensures that $\Delta\alpha$ is the rms spread in

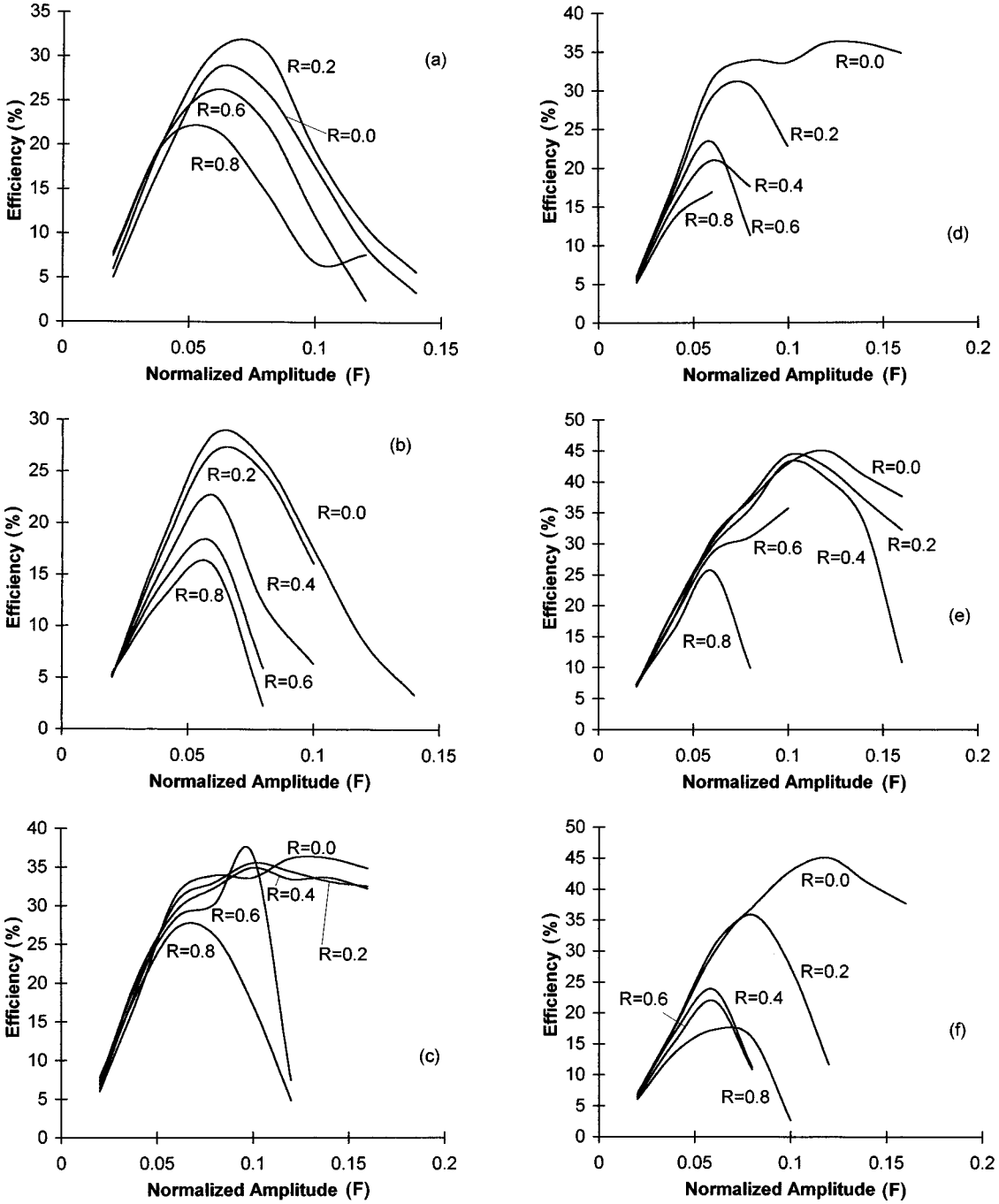

FIG. 3. Operation at a mode with two axial variations: (a), (c), and (e) correspond to the symmetric $TE_{m,p}$ mode ($m=0$); (b), (d), and (f) correspond to nonsymmetric ($m\neq 0$) modes. The bunching parameters for (a) and (b), (c) and (d), and (e) and (f) are equal to 0. 1.0, and 1.8, respectively.

pitch ratio. The results of calculations done for the 10% spread have shown that a double resonance operation at symmetric modes is as sensitive to the velocity spread as a single resonance interaction. For instance, the efficiency of the double resonance operation at a mode with two axial variations excited by a nonprebunched electron beam decreases from 24% to 19% [$R=0.2$; cf. Fig. 2(a)] while the efficiency of the single resonance operation in the same device with $R=0$ decreases from 21.2% to 15.6%. On the contrary, the double resonance operation at nonsymmetric modes is much more sensitive to the spread: for the same parameters ($l=2$, $q=0$, $R=0.2$) the 10% spread decreases the efficiency from 19.1% to 12.4%.

Now let us discuss the relation between the optimal field amplitude F (which can be found from Figs. 2 and 3) and the rf breakdown field. The normalized amplitude F used in Eqs. (17)–(20) is equal to

$$J_1(k_\perp R_g)\,\frac{1}{4\kappa}\,\frac{eA}{m_c c\omega}, \tag{22}$$

and its optimal value, as follows from Figs. 2–4, is on the order of 0.1. (Depending on other parameters, it varies approximately from 0.05 to 0.15.) The normalized axial wave number h in the case of the double resonance with a 500 kV,

$\alpha=1.5$ electron beam is equal to 0.694 ($h\simeq 1/3\beta_{z0}$). Correspondingly, the normalized transverse wave number $\kappa=\sqrt{1-h^2}$ is equal to 0.72. Let us also assume that the beam radius R_g corresponds to the maximum of the Bessel function $J_1(k_\perp R_g)\simeq 0.58$. This gives for the last term in Eq. (22) $eA/m_c c\omega\simeq 0.5$. From here the optimal field amplitude equals

$$A_{\rm opt}({\rm MV/m})\simeq\frac{160}{\lambda({\rm cm})}. \tag{23}$$

At high enough frequency ($f\gtrsim 10$ GHz) the rf breakdown field in the continuous wave (cw) regime can be approximated (see, e.g., Ref. [17]) by a simple expression:

$$E_{\rm br,cw}({\rm MV/m})\simeq 25\sqrt{f({\rm GHz})}\simeq\frac{137}{\sqrt{\lambda({\rm cm})}}. \tag{24}$$

From here it follows that the optimal amplitude does not exceed the breakdown field only at wavelengths longer than 1.36 cm. Note, however, that in many experiments it was shown that in pulse operation the breakdown field is much larger than in the cw regime. This allowed Wilson [18] to approximate the dependence of the breakdown field on the pulse duration τ (μs) by the following equation:

$$E_{\rm br}=E_{\rm br,cw}\left(1+\frac{4.5}{\tau^{1/4}}\right). \tag{25}$$

So, for instance, for $\tau\lesssim 1$ μs (which is a typical pulse duration of microwave sources intended for driving the linear colliders) the breakdown field is more than five times larger than that given by Eq. (24). This pushes the boundary of the wavelength region of stable operation to short millimeters. Note also that near the wall surface the electric field of symmetric modes is much weaker than in the interaction region that mitigates the breakdown problem.

For practical reasons, it makes sense also to estimate the resonator Q factor required for the generation of a certain microwave power, $P_{\rm rf}$, with the efficiency calculated above. Let us assume that the device should produce a 100 MW output power, which is the goal of the present program for the development of relativistic gyroklystrons for future linear colliders [7,19]. (For 500 kV beam voltage and 40% efficiency it implies a 500 A beam current.) In the steady state, the rf power extracted from the beam is equal to the power of microwave losses $(\omega/2Q)A^2N$, where N is the norm of the operating mode and the losses are mainly determined by the diffraction of radiation into the output waveguide. Therefore,

$$Q=\frac{\omega}{2}\frac{A^2N}{P_{\rm rf}}.$$

To calculate the norm, one should take into account the expressions for the electric and magnetic fields given in the Appendix by Eq. (A1) and the fact that the normalized axial and transverse wave numbers are determined by the double resonance condition. For the lowest symmetric modes in a cylindrical cavity (TE_{01l} and TE_{02l}) it gives $N_{01}=2.76\times 10^{-2}L\lambda^2$ and $N_{02}=5.2\times 10^{-2}L\lambda^2$. (In a similar manner one can also calculate the norms of the modes in coaxial cavities [20].) Then, using Eq. (23) for the optimal field amplitude one readily gets the following estimates for the required Q factors: $Q_{01l}\simeq 77l$, $Q_{02l}\simeq 145l$.

Before closing this section, let us also briefly discuss the issue of mode competition. Certainly, in the case of operation at the lowest symmetric modes the spectrum of modes differing in transverse indices is well rarefied (in contrast to high-power, long-pulsed gyrotron oscillators; see, e.g., Ref. [4]). However, the issue can be a competition between modes having the same transverse structure but different axial indices l when l is large. To evaluate the frequency separation between such modes let us consider the case studied above: a mode with three axial variations is excited by a 500 kV, $\alpha=1.5$ electron beam. As one can find using Eq. (2), the frequency of this mode is $1.39\omega_c$ (where ω_c is the cutoff frequency) while the frequencies of modes with two and four axial variations are equal to $1.19\omega_c$ and $1.63\omega_c$, respectively. So, the frequency of the closest mode is about 14% apart from the operating frequency. At the same time, the self-excitation band being estimated as $(\pi/2)T^{-1}$ [1] (where T is the electron transit time introduced in Sec. I) is smaller than 6%. This allows us to conclude that the mode competition should not be a severe problem for double resonance operation.

IV. SUMMARY

The formalism describing the simultaneous double cyclotron resonance interaction at different harmonics between the beam of gyrating electrons and one resonator mode was developed. It was shown that in the case of operation at azimuthally symmetric modes the efficiency of the double resonance interaction can be higher than that in the standard case of a single resonance interaction. For instance, for an optimally prebunched electron beam (one-cavity prebunching) it can reach 46%. In contrast, in the case of operation at nonsymmetric $TE_{m,p}$ modes (with $m\neq 0$) the additional resonance plays a destructive role only, especially when the velocity spread is significant. The estimates show that the optimal amplitude of the resonator field required for efficient operation is on the order of the breakdown field. Nevertheless, at least for a short pulse operation, the optimal field at all reasonable frequencies is smaller than the breakdown limit.

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APPENDIX

Here, we derive equations for electron motion in CRMs in the case of double resonance at different cyclotron harmonics. A single frequency electromagnetic field of a cavity can be represented as

$$\vec{E}={\rm Re}(A\vec{E}_s e^{i\omega t}),\quad \vec{H}={\rm Re}(A\vec{H}_s e^{i\omega t}),$$

where A is the field amplitude and functions $\vec{E}_s(\vec{r})$ and $\vec{H}_s(\vec{r})$ describe the spatial distribution of, respectively, the electric and magnetic fields. These functions can be expressed via the potential function $\Phi(\vec{r})$, which in turn can be represented in a weakly irregular cylindrical cavity as $\Phi(\vec{r}) = \Psi(\vec{r}_\perp) f(z)$. Here, $\Psi(\vec{r}_\perp)$ is the membrane function that obeys the Helmholtz equation $\Delta_\perp \Psi + k_\perp^2 \Psi = 0$ with the corresponding boundary condition and $f(z)$ describes the axial structure of the resonator field. In these notations the fields of any TE mode can be represented as

$$\vec{E}_s = \frac{k}{k_\perp^2} [\vec{\nabla}_\perp \Psi \times \vec{z}_0] f, \quad \vec{H}_s = i \left(\Psi f \vec{z}_0 + \frac{1}{k_\perp^2} \vec{\nabla}_\perp \Psi \frac{df}{dz} \right). \tag{A1}$$

The equation for electron motion can be written in normalized variables $z' = \omega z/c$, $p' = p/mc$, $A' = eA/mc\omega$, $\mu = eH_0/mc\omega$ (primes will be omitted below) as

$$\frac{d\vec{p}}{dz} + \mu \left[\frac{\vec{p}}{p_z} \times \vec{z}_0 \right] = -\frac{1}{\beta_z} \mathrm{Re} \left\{ A e^{i\omega t} \left(\vec{G}_1 f + \vec{G}_2 \frac{df}{dz} \right) \right\}. \tag{A2}$$

Here, functions $\vec{G}_1$ and $\vec{G}_2$, describing the spatial structure of the rf Lorentz force acting on electrons, in accordance with Eq. (A1), are equal to

$$\vec{G}_1 = \frac{1}{\kappa^2} [\vec{\nabla}_\perp \Psi \times \vec{z}_0] + i \Psi [\vec{\beta} \times \vec{z}_0], \tag{A3}$$

$$\vec{G}_2 = \frac{i}{\kappa^2} [\vec{\beta} \times \vec{\nabla}_\perp \Psi]. \tag{A4}$$

Here $\kappa = k_\perp/k$ is the transverse wave number $k_\perp$ normalized to $k = \omega/c$, $\vec{\beta} = \vec{v}/c$ is the electron velocity normalized to the speed of light and coordinates in the gradient $\vec{\nabla}_\perp$ are also normalized to ω/c.

The membrane function Ψ (when it is an analytic function) can be represented at any point with transverse coordinates X and Y as the superposition of harmonics of an angular variable θ:

$$\Psi = \sum_n \Psi_n(X, Y, r) e^{-in\theta}. \tag{A5}$$

In Eq. (A5) it is assumed that we use the reference frame with polar coordinates r and θ the center of which is located at the point (X, Y). As shown by Yulpatov [21], the coefficients Ψ_n in Eq. (A5) can be represented as

$$\Psi_n = J_n(\kappa r) L_n(X, Y), \tag{A6}$$

where

$$L_n = \left[\frac{1}{\kappa} \left(\frac{\partial}{\partial X} + i \frac{\partial}{\partial Y} \right) \right]^n \Psi(X, Y). \tag{A7}$$

Correspondingly, for an electron with transverse coordinates

$$x = X + a \cos\theta, \quad y = Y + a \sin\theta, \tag{A8}$$

where a is the Larmor radius, $\theta = \int_0^\tau \Omega d\tau' + \varphi$ is the gyrophase, Ω is the cyclotron frequency, $\tau = t - t_0$ is the transit time for an electron entering the cavity at the instant t_0, and X and Y are transverse coordinates of the guiding center, the components of functions $\vec{G}_1$ and $\vec{G}_2$ determined by Eqs. (A3) and (A4), respectively, can be defined as

$$G_{1,r} = i \sum_n J_n(\rho) L_n \left(\beta_\perp - \frac{n}{\kappa\rho} \right) e^{-in\theta}, \tag{A9a}$$

$$G_{1,\theta} = -\frac{1}{\kappa} \sum_n J_n'(\rho) L_n e^{-in\theta}, \tag{A9b}$$

$$G_{1,z} = 0, \tag{A9c}$$

$$G_{2,r} = -\frac{\beta_z}{\kappa\rho} \sum_n n J_n(\rho) L_n e^{-in\theta}, \tag{A9d}$$

$$G_{2,\theta} = \frac{i\beta_z}{\kappa} \sum_n J_n'(\rho) L_n e^{-in\theta}, \tag{A9e}$$

$$G_{2,z} = -\frac{i\beta_\perp}{\kappa} \sum_n J_n'(\rho) L_n e^{-in\theta}, \tag{A9f}$$

Here, $\rho = k_\perp a$ is the Larmor radius normalized to the transverse wave number.

Now these expressions for the components of $\vec{G}_1$ and $\vec{G}_2$ can be substituted into the right-hand side of Eq. (A2) and then this equation can be averaged over fast electron gyrations. To do this it is also necessary to specify the axial distribution of the fields since the Doppler term is important for the cyclotron resonance conditions given by Eq. (1). Below, we will consider the resonator with a sinusoidal axial field structure $f(z) = \sin(l\pi z/L) = \sin(hz)$ where $h = k_z/k = l\lambda/2L$ is the axial wave number normalized to k. Our consideration will be focused on the most practical case of the cyclotron resonance with the forward wave component of this standing wave at the fundamental harmonic and with the opposite wave component at the second harmonic, although the formalism used can be applied for arbitrary resonance. Corresponding cyclotron resonance conditions can be written in normalized variables as

$$1 - h\beta_z \approx \mu\gamma, \quad 1 + h\beta_z \simeq 2\mu\gamma. \tag{A10}$$

From Eq. (A10) one can readily derive the relation between h and β_z required for the double resonance, $h\beta_z \approx 1/3$, as well as other relations discussed in the Introduction. Using these cyclotron resonance conditions one can average the rf Lorentz force,

$$\vec{F} = -\frac{1}{\beta_z} \mathrm{Re} \left\{ A e^{i\omega t} \left(\vec{G}_1 f + \vec{G}_2 \frac{df}{dz} \right) \right\},$$

over fast electron gyrations and get as a result only two terms proportional to slowly variable phases $\vartheta_1 = \omega t - k_z z - \theta$, $\vartheta_2 = \omega t + k_z z - 2\theta$:

$$\langle \vec{F} \rangle = -\frac{1}{2\beta_z} \mathrm{Re}\{iA[e^{i\vartheta_1}(\vec{G}_1 - ih\vec{G}_2)_1 - e^{i\vartheta_2}(\vec{G}_1 + ih\vec{G}_2)_2]\}. \quad \text{(A11)}$$

Here, the angular brackets denote the averaging over fast gyrations (these brackets will be omitted below) and indices ''1'' and ''2'' in $(\vec{G}_1 \mp ih\vec{G}_2)$ denote corresponding components in the superposition $\Sigma_n(\vec{G}_1 \mp ih\vec{G}_2)_n e^{-in\theta}$.

Using Eq. (A8) for transverse coordinates of electrons and representing the transverse components of their momentum as

$$p_x = -p_\perp \sin\theta, \quad p_y = p_\perp \cos\theta$$

one can derive by applying the Van der Pol method for electron momentum and coordinates of guiding centers the following set of equations:

$$\frac{dp_\perp}{dz} = F_\theta, \quad p_\perp \frac{d\varphi}{dz} = -F_r, \quad \frac{dp_z}{dz} = F_z \quad \text{(A12)}$$

and

$$\mu \frac{dX}{dz} = -F_y, \quad \mu \frac{dY}{dz} = F_x. \quad \text{(A13)}$$

Here the components of the rf Lorentz force averaged over fast gyrations, in accordance with Eqs. (A9) and (A11), are equal to

$$F_\theta = \frac{1}{3\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} - 2J_2'(\rho)L_2 e^{i\vartheta_2}]\}, \quad \text{(A14a)}$$

$$F_r = -\frac{1}{3\kappa\beta_z} \mathrm{Re}\{A[[\rho J_1'(\rho)]' L_1 e^{i\vartheta_1} - [\rho J_2'(\rho)]' L_2 e^{i\vartheta_2}]\}, \quad \text{(A14b)}$$

$$F_z = \frac{h\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} + J_2'(\rho)L_2 e^{i\vartheta_2}]\}, \quad \text{(A14c)}$$

$$F_y = -\frac{\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\left\{A\left[J_1'(\rho)\frac{\partial L_1}{\partial Y} e^{i\vartheta_1} - J_2'(\rho)\frac{\partial L_2}{\partial Y} e^{i\vartheta_2}\right]\right\}, \quad \text{(A14d)}$$

$$F_x = -\frac{\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\left\{A\left[J_1'(\rho)\frac{\partial L_1}{\partial X} e^{i\vartheta_1} - J_2'(\rho)\frac{\partial L_2}{\partial X} e^{i\vartheta_2}\right]\right\}. \quad \text{(A14e)}$$

Note that, as was done by Yulpatov for gyrotron oscillators operating at frequencies near the cutoff (see also Ref. [22] for operation at traveling waves), we used in derivation of the expression for F_r the Bessel equation

$$\left(\rho - \frac{n^2}{\rho}\right) J_n(\rho) = -[\rho J_n'(\rho)]'$$

and in derivation of the expressions for F_x and F_y, first, the recurrent relation

$$J_{n\pm1}(\rho) = \frac{n}{\rho} J_n(\rho) \mp J_n'(\rho),$$

and second, the relation

$$L_{n+1} \pm L_{n-1} = \frac{1}{\kappa}\left[\left(\frac{\partial}{\partial X} + i\frac{\partial}{\partial Y}\right) \mp \left(\frac{\partial}{\partial X} - i\frac{\partial}{\partial Y}\right)\right] L_n = \frac{2}{\kappa}\begin{Bmatrix} i\frac{\partial}{\partial Y} \\ \frac{\partial}{\partial X} \end{Bmatrix} L_n.$$

Also note that using the condition of the double resonance one can express in Eq. (A14) the components of the wave vector via the components of initial electron velocity:

$$h \simeq \frac{1}{3}\beta_{z0}, \quad \kappa = \frac{\sqrt{9\beta_{z0}^2 - 1}}{3\beta_{z0}}.$$

It is also expedient to augment Eqs. (A12) and (A13) with the equation for the normalized electron energy γ:

$$\frac{d\gamma}{dz} = \frac{\beta_\perp}{2\kappa\beta_z} \mathrm{Re}\{iA[J_1'(\rho)L_1 e^{i\vartheta_1} - J_2'(\rho)L_2 e^{i\vartheta_2}]\}. \quad \text{(A15)}$$

As follows from the equations for electron energy and axial momentum, in the case of the double resonance the autoresonance integral is no longer valid. Recall that in the case of the resonance with the forward wave it has the form [14,23] $p_z - h\gamma = \text{const}$ and for the resonance with the opposite wave [14,24], $p_z + h\gamma = \text{const}$.

In the case of the double resonance it is also impossible to derive the same simple relation between the radial drift of electron guiding centers and changes in the electron energy as in the case of a single resonance. (This relation, first derived by Yulpatov for gyrotron oscillators, was then given for gyro-traveling wave tubes in Ref. [22].) Now, after representing the coordinates of the guiding centers are $X = R_g \cos\psi$, $Y = R_g \cos\psi$, one can derive for R_g the following equation:

$$R_g \frac{dR_g}{dz} = \frac{\beta_\perp}{2\kappa\mu\beta_z} \mathrm{Re}\{iA[(1 \mp m)L_1 J_1'(\rho)e^{i\vartheta_1} - (2 \mp m)L_2 J_2'(\rho)e^{i\vartheta_2}]\} \quad \text{(A16)}$$

So, the difference in resonant harmonic number (''1'' for the first term in the right-hand side and ''2'' for the last term) makes the right-hand side of this equation different from the right-hand side of Eq. (A15) for γ. However, we can estimate the effect of each wave on the radial drift separately and this leads us to the conclusion that was done in Ref. [22] for a single resonance: when a number of electron orbits in the interaction region is large, $N \gg 1$, the radial drift of the electron guiding centers is negligibly small. Recall that this condition, $N \gg 1$, is also necessary for averaging the equations for electron motion over fast gyrations. Therefore, later we will not take into account the drift of electron guiding centers.

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